### 

### Semester One Examination, 2022

### Question/Answer booklet

# MATHEMATICS METHODS

**UNIT 3**

## Section One:

## Calculator-free

|  |  |
| --- | --- |
| **Your Name:** |  |
| **Your Teacher’s Name:** |  |

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Question | Marks | Max | Question | Marks | Max |
| 1 |  | 8 | 5 |  | 6 |
| 2 |  | 10 | 6 |  | 13 |
| 3 |  | 8 |  |  |  |
| 4 |  | 8 |  |  |  |

**Structure of this paper**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 6 | 6 | 50 | 53 | 35 |
| Section Two:  Calculator-assumed | 12 | 12 | 100 | 100 | 65 |
|  |  |  |  | **Total** | 100 |

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.**Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

**Section One: Calculator-free (53 marks)**

This section has **six** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

● Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.

● Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 (8 marks)

The function is defined for by , and .

1. Show that (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| ✓ Shows the use of the quotient rule to determine  ü Correctly determines |

1. Determine the coordinates and nature of all stationary points of . Justify your answer. (4 marks)

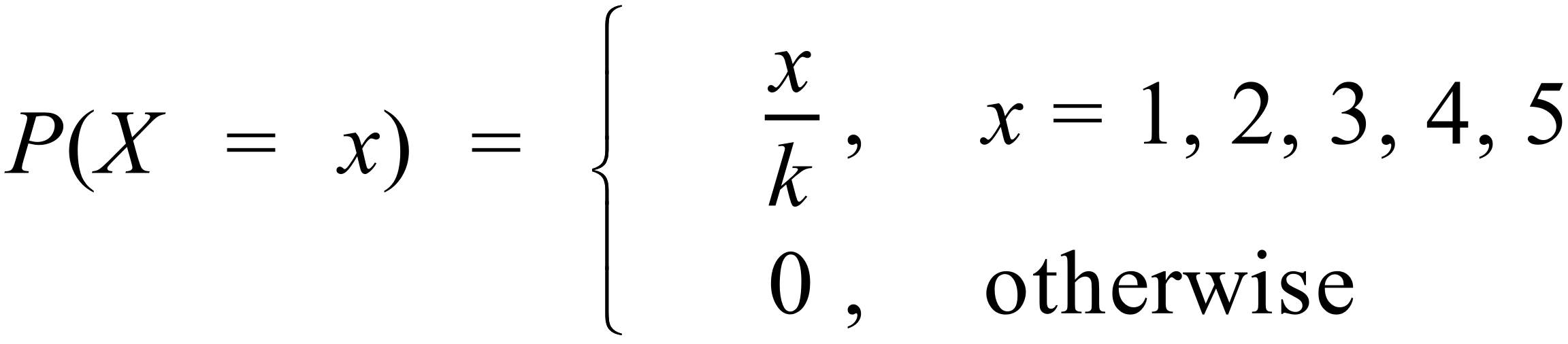
|  |
| --- |
| Solution |
| The stationary point at is a MINIMUM  The minimum turning point of the function occurs at |
| Specific Behaviours |
| ü Equates to zero, and solves for the -coordinate of the stationary point (only )  ü Uses the sign test or second derivative test to justify the nature of the -coordinate  ü Correctly evaluates and interprets the nature of the stationary point  ü Correctly determines the coordinates of the stationary point |

1. Show that has no points of inflection. (2 marks)

|  |
| --- |
| Solution |
| If there are points of inflection:  But if we use discriminants to obtain solutions:  Since this shows that we have no solutions from the quadratic. Therefore, there will be no points of inflection to the function. |
| Specific Behaviours |
| ü Uses (must equate the actual function for the second derivative to 0)  ü Uses the quadratic to explain why there are no points of inflection (either by discriminants, completing the square, or the quadratic formula) |

**Question 2 (10 marks)**

The discrete random variable has a probability function with



(a) Determine the value of (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| ✓ Uses the sum of probabilities equal to 1 to establish an expression for finding  ü Determines the value of |

Determine:

(b) (i) (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| ✓ Shows the correct numerator  ü Determines the correct conditional probability |

(ii) (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| ✓ Shows the sum of each score with their respective probability  ü Determines |

(c) A second discrete random variable is defined to be

If and the standard deviation of is, determine and . (4 marks)

|  |
| --- |
| Solution |
| and and  Setting up two equations to solve for and :  or  or  If using the standard deviation, the correct second equation is: |
| Specific Behaviours |
| ✓ Sets up one correct equation to solve for and  ü Sets up two correct equations to solve for and  ✓ Solves for one pair of values for and  ü Solves for both pairs of values for and |

Question 3 (8 marks)

<EFOFEX>
id:fxd{ff8b9e87-281b-4421-9a2e-8ead16b06f94}

FXData:
</EFOFEX>The graph of between  
 and is shown at right.  
  
Approximate values for and   
are and respectively.

(a) Use the areas of the rectangles shown to explain why . (3 marks)

|  |
| --- |
| Solution |
| The value of the integral is the area under the curve between and . The area of the inscribed rectangles is , an underestimate. The area of the circumscribed rectangles is , an overestimate. Hence the value of the integral must lie between these two. |
| Specific Behaviours |
| ✓ Derives area approximation using inscribed rectangles  ü Derives area approximation using circumscribed rectangles  ü Explains inequality  (If used surd form, 1 mark deducted) |

(b) Evaluate . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| ü Obtains term in antiderivative  ü Obtains correct antiderivative  ü Substitutes both bounds and simplifies  (1 mark deducted, if answer was given in surd form ) |

(c) Evaluate . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| ✓ Uses linearity  ü Correct value |

Question 4 (8 marks)

A student observes the graphs of three binomial distributions with the assigned random variables and for each graph. For all three distributions, the value of is constant, but the values of are 0.25, 0.5 and 0.9 respectively.

1. The student notes that one of the distributions produced a graph that is skewed to the right (long tail to the right). Which of the values of were used to produce this graph? Give justification for your choice. (2 marks)

|  |
| --- |
| Solution |
| A lower value for the probability of success will skew the data to the right as this will decrease the overall expected value. |
| Specific Behaviours |
| * States the correct value of * Justifies with regard to the expected value of the distribution |

1. The standard deviation when is 6. Determine the value of . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| * Determines the variance of the distribution * Establishes an equation to solve using the variance * Solves for (accept correct integer or fraction value) |

1. Using your answer from part b), show how to calculate the following without the use of a calculator. Do not evaluate your answer.
   1. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| * Gives the correct expression for determining |

* 1. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| * Shows the sum of probabilities * Gives the correct expression for determining |

**Question 5 (6 marks)**

1. Determine (2 marks)

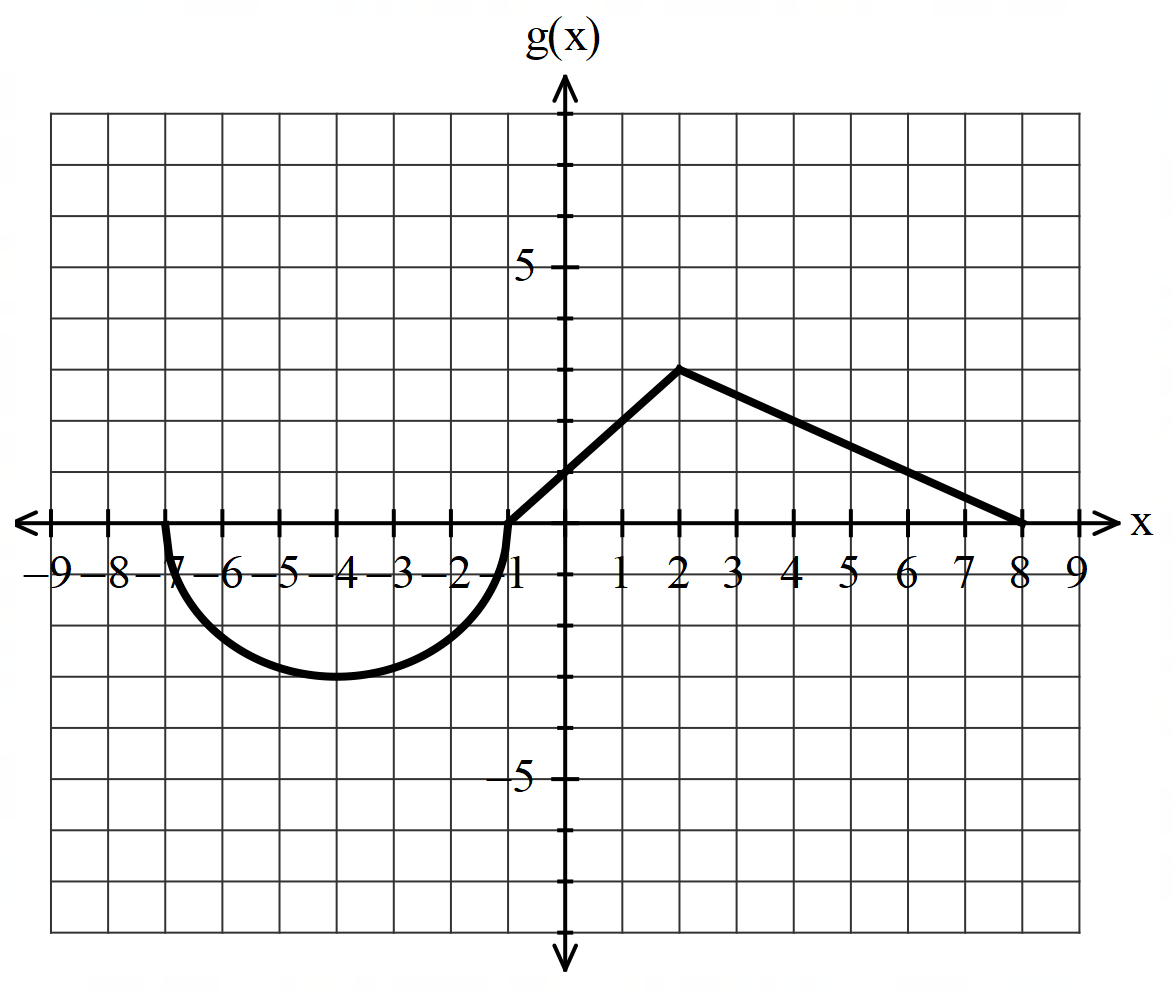
|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| * Demonstrates the use of the chain rule * Determines the correct expression for the derivative |

1. Hence, evaluate the following in exact form: (4 marks)

|  |
| --- |
| Solution |
| By FTC: |
| Specific Behaviours |
| * Uses FTC to establish the antiderivative function * Determines the antiderivative function * Substitutes the limits * Evaluates the integral exactly |

**Question 6 (13 marks)**

The graph of is given as below, which consists of a semi-circle for and a triangle for



Given that , where .

1. Determine the intervals where is increasing and decreasing, respectively.

(2 marks)

|  |
| --- |
| **Solutions** |
| Increasing at and decreasing at |
| **Specific Behaviours** |
| * Determines the correct increasing interval. * Determines the correct decreasing interval. |

1. Determine the intervals where is concave up and concave down, respectively.

(2 marks)

|  |
| --- |
| **Solutions** |
| Concave up at concave down at and |
| **Specific Behaviours** |
| * Determines the correct concave up interval. * Determines the correct concave down intervals |

1. Determine the value(s) of when reaches any stationary point(s) (2 marks)

|  |
| --- |
| **Solutions** |
| stationary at |
| **Specific Behaviours** |
| * Determines at x=-1 * Determines at x=-7 & 8 |

1. Determine the exact values of . (2 marks)

|  |
| --- |
| **Solutions** |
|  |
| **Specific Behaviours** |
| * Uses area under the curve. * Coverts into negative integral. |

1. Determine the exact values . (2 marks)

|  |
| --- |
| **Solutions** |
|  |
| **Specific Behaviours** |
| * Determines the correct area of triangle. * Determines the correct integral. |

1. Sketch the graph of in the axes provided below, for Label key features. (3 marks)

Chart, scatter chart

Description automatically generated

|  |
| --- |
| **Solutions** |
| Note: Students sketch based on concavity and information above- exact graph on classpad is for teacher use only |
| **Specific Behaviours** |
| * shape with correct concavity   √ Labels local min with coords\_and plotted accurately (accept approx.)   * Labels endpoints with coords and plotted accurately (accept approx.) |

Additional working space

Question number: \_\_\_\_\_\_\_\_

Additional working space

Question number: \_\_\_\_\_\_\_\_